3.0 CONCRETE STRUCTURES

3.1 Material Properties

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3.9 Design Example. AASHTO-PCI Bulb-Tee BT-72, Single Span With Composite Deck, Designed According to AASHTO LRFD Specifications

3.0 CONCRETE STRUCTURES

3.1 Material Properties. (LRFD Art. 5.4)

3.1.1 Creep (LRFD Art. 5.4.2.3.2)

The Creep Coefficient

$$\psi(t,t_i) = 1.9 k_s k_{hc} k_f k_{td} t_i^{-0.118} \qquad \text{(LRFD Eq. 5.4.2.3.2-1)}$$

for which:

$$k_s = 1.45 - 0.13(V/S) \geq 1.0 \qquad \text{(LRFD Eq. 5.4.2.3.2-2)}$$

$$k_{hc} = 1.56 - 0.008H \qquad \text{(LRFD Eq. 5.4.2.3.2-3)}$$

$$k_f = \frac{5}{1 + f'_{ci}} \qquad \text{(LRFD Eq. 5.4.2.3.2-4)}$$

$$k_{td} = \left(\frac{t}{61 - 4f'_{ci} + t}\right) \qquad \text{(LRFD Eq. 5.4.2.3.2-5)}$$

3.1.2 Shrinkage (LRFD Art. 5.4.2.3.3)

The strain due to shrinkage:

$$\varepsilon_{sh} = k_s k_{hs} k_f k_{td} 0.48 \times 10^{-3} \qquad \text{(LRFD Eq. 5.4.2.3.3-1)}$$

in which:

$$k_{hs} = (2.00 - 0.014H) \qquad \text{(LRFD Eq. 5.4.2.3.3-2)}$$

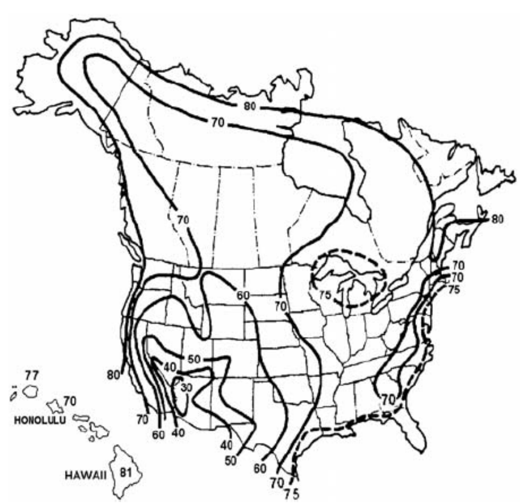

Fig 3-1 Annual Average Ambient Relative Humidity in Percent (LRFD Fig 5.4.2.3.3-1)

3.1.3 Modulus of Elasticity & Rupture of Concrete (LRFD Art. 5.4.2.4&6)

Modulus of Elasticity E_c

$$E_c = 33{,}000 K_1 w_c^{1.5} \sqrt{f_c'}$$ (LRFD Eq. 5.4.2.4-1)

Modulus of Rupture

- For normal weight concrete:
  - When used to calculate the cracking moment of a member in Articles 5.7.3.4 and 5.7.3.6.2………… $0.24\sqrt{f_c'}$
  - When used to calculate the cracking moment of a member in Articles 5.7.3.3.2 ……………………... $0.37\sqrt{f_c'}$
  - When used to calculate the cracking moment of a member in Articles 5.8.3.4.3 ……………….…….. $0.20\sqrt{f_c'}$
- For light weight concrete:
  - For sand-lightweight concrete………… $0.20\sqrt{f_c'}$
  - For all-lightweight concrete …………… $0.17\sqrt{f_c'}$

3.1.4 Prestressing Steel

Material	Grade or Type	Dia. IN	Tensile Strength, f_{pu} (KSI)	Yield Strength, f_{pu} (KSI)
Strand	250 KSI 270 KSI	1/4 to 0.6 3/8 to 0.6	250 270	85% of f_{pu}, except 90% of f_{pu}, for low-relaxation strand
Bar	Type 1, Plain Type 2, Deformed	3/4 to 1-3/8 5/8 to 1-3/8	150 150	85% of f_{pu} 80% of f_{pu}

Table 3-1 Properties of Prestressing Strand and Bar (LRFD Table 5.4.4.1-1)

3.2 Fatigue Limit State (LRFD Art. 5.5.3)

- Reinforcing Bars

  without a cross weld in the high-stress region:

  $f_f = 24 - 0.33 f_{min}$ (LRFD Eq 5.5.3.2-1)

  with a cross weld in the high-stress region:

  $f_f = 16 - 0.33 f_{min}$ (LRFD Eq 5.5.3.2-2)

- Prestressing Tendons

  18ksi for R>30′

  10ksi for R>12′

3.3 Strength Limit State (LRFD Art. 5.5.4)

- Conventional Construction:

  Resistance Factors ϕ:

  - For tension-controlled reinforced concrete sections. 0.90
  - For tension-controlled prestressed concrete sections 1.00
  - For shear and torsion:
    - normal weight concrete...0.90
    - lightweight concrete..0.70
  - For compression-controlled sections with spirals or ties, as defined in Article 5.7.2.1, except as specified in Article 5.10.11.4.1b for Seismic Zones 3 and 4 at the extreme event limit state..0.75
  - For bearing on concrete..0.70
  - For compression in strut-and-tie models 0.70
  - For compression in anchorage zones:
    - normal weight concrete...0.80
    - lightweight concrete..0.65
  - For tension in steel in anchorage zones...1.00
  - For resistance during pile driving...1.00

- For a prestressed members:

$$0.75 \le \phi = 0.583 + 0.25\left(\frac{d_t}{c} - 1\right) \le 1.0 \qquad \text{(LRFD Eq 5.5.4.2.1-1)}$$

- For nonprestressed members:

$$0.75 \le \phi = 0.65 + 0.15\left(\frac{d_t}{c} - 1\right) \le 0.9 \qquad \text{(LRFD Eq 5.5.4.2.1-2)}$$

- For tension-controlled partially prestressed components in flexure:

$$\phi = 9.0 + 0.1(PPR) \qquad \text{(LRFD Eq 5.5.4.2.1-3)}$$

in which:

$$PPR = \frac{A_{ps} f_{py}}{\left(A_{ps} f_{py} + A_s f_y\right)} \qquad \text{(LRFD Eq. 5.5.4.2.1-4)}$$

- Segmental Construction

	ϕ_f Flexure	ϕ_v Shear
Normal Weight Concrete		
Fully Bonded Tendons	0.95	0.90
Unbonded or Partially Bonded Tendons	0.90	0.85
Sand-Lightweight Concrete		
Fully Bonded Tendons	0.90	0.70
Unbonded or Partially Bonded Tendons	0.85	0.65

Table 3-2 Resistance Factor for Joints in Segmental Construction.

3.4 Flexure

3.4.1 Stages of Loading

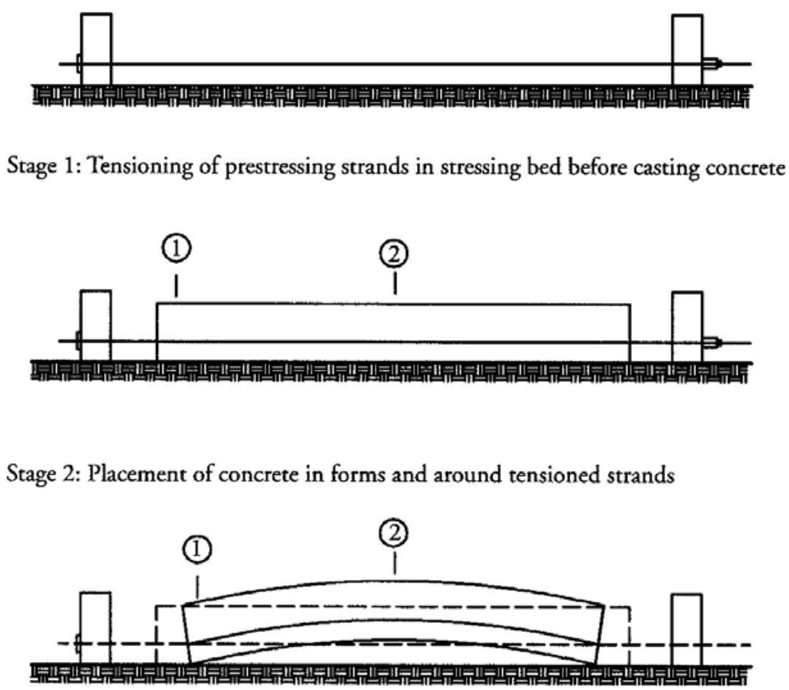

Figure 3-2 (PCI Fig. 8.2.1.1-1) Loading Stages of a Precast Prestressed Concrete Bridge Beam

3.4.2 Allowable Concrete Stress by LRFD (Other than segmentally constructed bridges)

Stress limits for concrete at release (LRFD Art. 5.9.4.1):

1. Compression for pretensioned or post-tensioned members, $0.60 f'_{ci}$

2. Tension:
   - a) in precompressed tensile zone without. bonded reinforcement, N/A
   - b) in areas other than the precompressed tensile zone and without bounded reinforcement, $0.0948\sqrt{f'_{ci}} \leq 0.2$ ksi
   - c) in areas with bounded reinforcement (reinforcing bars or prestressing steel) sufficient to resist the tensile force in the concrete computed assuming an uncracked section, where reinforcement is proportioned using a stress of $0.5 f_y$, not to exceed 30 ksi, $0.24\sqrt{f'_{ci}}$ ksi
   - d) for handling stresses in prestressed piles, $0.158\sqrt{f'_{ci}}$ ksi

Stress limits for concrete at Service Limit State for fully prestressed components (LRFD Art. 5.9.4.2):

1. Compression using the service limit state Load Combination I:
   a) due to the sum of effective prestress and permanent (dead) load, (i.e. beam self-weight, deck slab weight, diaphragm weight, wearing surface and barrier weights), $0.45f'_c$
   b) due to the sum of effective prestress, permanent and transient loads, i.e. all dead loads and live loads, and during shipping and handling, $0.60\phi_w f'_c$
   c) In other than segmentally constructed bridges due to live load and one-half of the sum of effective prestress and permanent loads, $0.40f'_c$

2. Tension using the service limit state Load Combination III, where only 80% of the live load effects are considered:
   a) for components with bonded prestressing tendons or reinforcement that are subjected to not worse than moderate corrosion condition, $0.19\sqrt{f'_c}$, ksi
   b) for components with bonded prestressing tendons or reinforcement that are subjected to serve corrosive conditions, $0.0948\sqrt{f'_c}$, ksi
   c) for components with unbonded prestressing, no tension is allowed

3.4.3 Design Procedure

Generally, the tensile stresses at midspan due to full dead and live loads plus effective prestress (after losses) controls the design.

1. Compute the tensile stress due to beam self-weight plus any other non-composite loads such as the deck, deck forms, haunches, diaphragms, etc., if any, applied to the beam section only.

2. Compute the tensile stress due to superimposed dead loads plus live load *(Standard Specifications*) or 0.8 live load (*LRFD Specifications*) applied to the composite section.

3. The net stress, f_b, due to loads in Steps 1 plus 2, minus the allowable tensile stress is the stress that needs to be offset by prestressing:

$$\frac{P_{se}}{A} + \frac{P_{se}e_c}{S_b}$$

   where P_{se} is the effective prestress, e_c is strand eccentricity at midspan, and A and S_b are beam area and bottom fiber modulus.
   Solve for P_{se}. The estimated number of stands P_{se} / (area of one strand) (f_{pe}), where f_{pe} is the effective prestress after all losses which may be approximated as 160 ksi for Grade 270 strand.

4. Perform a detailed calculation of prestress losses and repeat Step 3 if necessary.

5. Check stresses at the ends (transfer length) and midspan at release and at service. Check stresses at the harp point at release. Under typical load conditions, stresses at harp points do not govern at service loads and are therefore not checked. Determine the amount of harping and/or debonding required to control stresses at the end of the beam. This may be done by computing a required "e" for the selected P_{se} when draping is used, or by computing the required P_{se} for a given "e" when debonding is used.

6. Check strength.

7. If necessary, revise number of stands and repeat Steps 4 and 5.

3.4.4 Strand Considerations

3.4.4.1 Harped Strand

When concrete stresses exceed allowable limits, strand harping becomes an attractive option to reduce prestress eccentricity. The designer should be familiar with the practice and limitations of local producers when considering whether or not the calculated force and harp angle can be tolerated. The following are some options to consider if the hold-down force exceeds that which the fabricators can accommodate:

1. Split the strands into two groups with separate hold-downs.

2. Change slope of harp by moving harp points closer to centerline of the beam, or by lowering harp elevation at beam ends, or both. Also, consider uplift force and harp angle.

3. Decrease the number of harped strands.

4. Use debonding instead of harping or combine debonding with harping to reduce harping requirements.

3.4.4.2 Debonded Strand

An alternative to strand harping is to reduce the total prestress force by debonding some strands at the ends of members. After prestress is released to the concrete member, the debonded length of the strand has zero stress. Strand debonding may be more economical for some precast producers than harping. However, designers should take into account the effects of the reduction of precompression, (P/A), as well as the loss of the vertical component of prestress which contributes to shear resistance near the member ends. In addition, the calculated strand development length at the end of a debonded strand is required to be doubled by the *Standard Specifications*. Debonded strands have been shown by recent studies, Russell and Burns (1993, 1994-A and 1994-B), to perform well and their use is encouraged whenever possible. The *Standard Specifications* do not contain specific requirements regarding the maximum number and distribution of debonded strands. However, Article 5.11.4.2 of the *LRFD Specifications* provides the following rules if debonded prestressing strands are used:

1. The number of partially debonded strands should not exceed 25% of the total number of strands.

2. The number of debonded strands in any horizontal row shall not exceed 40% of the strands in that row.

3. Debonded strands should be symmetrically distributed about the centerline of the member.

4. Exterior strands in each horizontal row should be fully bonded.

However, these rules appear to be too conservative according to current practice in several states and the recent studies by Russell and Burns (1993, 1994-A and 1994-B), and others.

3.4.4.3 Minimum Strand Cover and Spacing

The *Standard Specifications* require a minimum concrete cover over strands of 1.50 in. The *LRFD Specifications* are unclear regarding concrete cover over prestressing stand in precast concrete beams. For precast soffit form panels (stay-in-place deck panels), the minimum cover is 0.80 in. and for members subject to exterior exposure, the minimum is 2.0 in. regardless of whether the member is precast or cast-in-place. It is recommended here to use the 1.50 in. minimum cover specified in the *Standard Specifications* for bridge beams.

The Federal Highway Administration has approved use of ½ in. diameter strand at a spacing of 1.75 in., and 0.6 in. diameter strand at 2.00 in. on center. As a result, box beams, for example, may have two layers of ½ in. diameter strands in the bottom flange using one of the alternative patterns. If the vertical strand spacing is desired to be 2 in., the bottom flange thickness may have to be increased to satisfy the minimum cover requirements.

3.4.5 Nominal Flexural Resistance

3.4.5.1 Required Parameters

The average stress in bonded prestressing steel, $f_{ps} = f_{pu}\left(1 - k\dfrac{c}{d_p}\right)$ (LRFD Eq. 5.7.3.1.1-1)

Assuming rectangular section behavior, the neutral axis depth:

$$c = \frac{A_{ps} f_{pu} + A_s f_s - A'_s f'_s}{0.85 f'_c \beta_1 b + k A_{ps} \dfrac{f_{pu}}{d_p}}$$ (LRFD Eq. 5.7.3.1.1-4)

where

c = distance between the neutral axis and the compressive face
A_{ps} = area of prestressing steel
f_{pu} = specified tensile strength of prestressing steel
A_s = area of mild steel tension reinforcement
f_s = stress in the mild steel tension reinforcement at nominal flexural resistance
A'_s = area of compression reinforcement
f'_s = stress in the mild steel compression reinforcement at nominal flexural resistance
b = width of compression of flange
k = factor related to type of strand:
= $2\left(1.04 - \dfrac{f_{py}}{f_{pu}}\right)$ (LRFD Eq. 5.7.3.1.1-2)
= 0.28 for low relaxation strand
f_{py} = yield strength of prestressing steel
d_p = distance from extreme compression fiber to the centroid of the prestressing strand

The depth of the compression block, $a = \beta_1 c$. If $a > h_f$ (depth of the compression flange), flanged section behavior must be used with c calculated by:

$$c = \frac{A_{ps} f_{pu} + A_s f_s - A'_s f'_s - 0.85 f'_c (b - b_w) h_f}{0.85 f'_c \beta_1 b_w + k A_{ps} \dfrac{f_{pu}}{d_p}}$$ (LRFD Eq. 5.7.3.1.1-3)

where b_w = width of web

3.4.5.2 Rectangular Sections

$$M_n = A_{ps} f_{ps}\left(d_p - \frac{a}{2}\right) + A_s f_s\left(d_s - \frac{a}{2}\right) - A'_s f'_s\left(d'_s - \frac{a}{2}\right)$$ (LRFD Eq. 5.7.3.2.2-1)

3.4.5.3 Flanged Sections

$$M_n = A_{ps} f_{ps}\left(d_p - \frac{a}{2}\right) + A_s f_s\left(d_s - \frac{a}{2}\right) - A'_s f'_s\left(d'_s - \frac{a}{2}\right) + 0.85 f'_c (b - b_w) h_f \left(\frac{a}{2} - \frac{h_f}{2}\right)$$

Where

f_{ps} = average stress in prestressing steel
a = depth of the equivalent stress block = ($\beta_1 c$)
A_s = area of non prestressed tension reinforcement
d_s = distance from extreme compression fiber to the centroid of nonprestressed tensile reinforcement
A'_s = area of compression reinforcement
d'_s = distance from extreme compression fiber to the centroid of nonprestressed compression reinforcement

Factored flexural resistance:

$$M_r = \phi M_n \qquad \text{(LRFD Eq. 5.7.3.2.1-1)}$$

Where ϕ = resistance factor = 1.00

3.4.6 Maximum and Minimum Reinforcement Limit

3.4.6.1 Maximum Limit

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3.4.6.2 Minimum Limit

Unless otherwise specified, at any section of a flexural component, ,the amount of prestressed and nonprestressed reinforcement should be adequate to developed a factored flexural resistance, M_r, at least equal to the lesser of 1.2 times the cracking strength determined on the basis of elastic stress distribution and the modulus of rupture, f_r, of the concrete, or 1.33 times the factored moment required by the applicable strength load combinations.

The *LRFD Specifications* give a similar procedure for computing the cracking moment, M_{cr}.

$$M_{cr} = S_c\left(f_r + f_{pce}\right) - M_{dnc}\left(\frac{S_c}{S_{nc}} - 1\right) \geq S_c f_r \qquad \text{(LRFD Eq. 5.7.3.3.2-1)}$$

where S_c, S_{nc} = composite and noncomposite section modulus, f_{cpe} = compressive stress in concrete due to effective prestress forces at extreme fiber of section; f_r = modulus of rupture = $0.37\sqrt{f'_c}$

Contrary to the *Standard Specifications*, the *LRFD Specifications* require that this criterion be met at all sections.

3.5 Flexural Design Examples

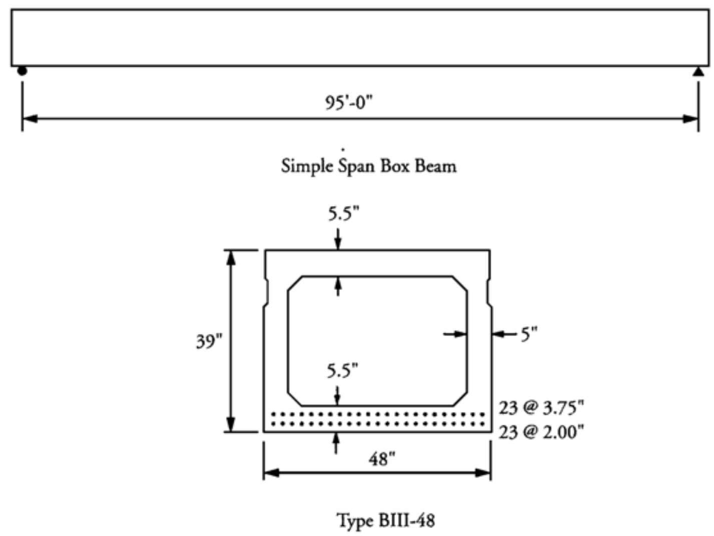

Figure 3-3

3.5.1 Design Example

Design Requirement 1

Does the midspan section have adequate flexural strength to resist a factored load moment, M_u = 4,900 kip-ft?

Standard Specifications

Assume the depth of the compression block, *a*, to fall within the top flange.

Compute the average stress in the prestressing steel at ultimate load, f_{su}^*, using STD Eq. 9-17 with, f_s' = 270 ksi, γ^* = 0.28, β_1 = 0.76, and:

$$\rho^* = \frac{A_s}{bd} = \frac{46(0.153)}{48.0(36.13)} = 0.00406$$

There for, $f_{su}^* = 270\left(1 - \frac{0.28(0.00406)(270)}{0.76(5.8)}\right) = 251$ ksi.

The compression block depth, $a = \frac{A_s^* f_{su}^*}{0.85 f_c' b} = \frac{46(0.153)(251)}{0.85(5.8)(48)} = 7.47$ in.

This is larger than the flange thickness (5.50 in.). Therefore, the section behaves as a flanged section,

with

b = 48.00 in.
b' = 2(5) = 10.00 in.
t = 5.50 in.
A_{sf} = $0.85 f'_c (b - b')/f^*_{su}$ = 4.110 in.^{2}

Thus $A_{sr} = A^*_s - A_{sf}$ = 2.928 in.^{2}

The corresponding steel index, $A_{sr} f^*_{su} / (b' d f'_c) = 0.35$. This exceeds the maximum steel index of $0.36\beta_1 = 0.36(0.76) = 0.27$. Thus, the section must be designed as an over-reinforced section. Using [STD Eq. 9-23], $\phi M_n = 4{,}301$ ft-kips. Note that when reinforcement amounts greater than the maximum limit are used, their effectiveness is significantly diminished. Such design is rare as it is generally uneconomical.

The design capacity, 4,301 ft-kips, is less than the required capacity of 4,900 ft-kips. The capacity may be improved by increasing the f'_c value. Increasing f'_c Would reduce the reinforcement index and improve the lever arm distance between the center of the strand group and the center of the compression block. Use value of f'_c = 8,500 psi. This significantly larger value than 5,800 psi was chosen for the purpose of comparison of the results with *LRFD Specifications* and strain compatibility solutions given later. The values of β_1 and f^*_{su} Become 0.65 and 255 ksi. The corresponding a = 5.18 in., which is less than 5.5 in. Therefore, ϕM_n = 5,009 Ft-kips which is acceptable. The steel index $\rho^* f^*_{su} = 0.12$ which is much lower than the limit $0.36\beta_1 = 0.23$. It should be noted that it is not unusual to have flexural strength rather than service stress control the design of adjacent box beam bridges.

LRFD Specifications

Use LRFD Eq. (5.7.3.1.1-3)

with

A_{ps} = 7.038 in.^{2}
f_{pu} = 270 ksi
β_1 = 0.76
f'_c = 5.8 ksi
$(b - b_w)$ = 38 in.
h_f = 5.50 in.
b_w = 10.00 in.
k = 2(1.04 – 0.9) = 0.28 and d_p = 36.13 in., the neutral axis depth c = 21.40 in. and c/d_p = 0.59. This is greater than the maximum value of 0.42. The section is over-reinforced and LRFD Eq. (C5.7.3.3.1-2), which is identical to *Standard Specifications* Eq. 9-23, must be used. The resulting ϕM_n would therefore be identical to that obtained earlier.

If the f_c' value is increased to 8.5 ksi, the neutral axis depth = 14.89 in., and c/d_p = 0.41 which is slightly less than the maximum value Thus, the section is under-reinforced and LRFD Eq. (5.7.3.2.2-1) may be used. Substituting into this equation with $a = \beta_1 c$ = 9.68 in., f_{ps} = 270(1 – 0.28(14.89)/36.13) = 239 ksi, ϕM_n = 4,557 ft-kips. This value is less than the capacity needed. Note that the values of a, f_{ps} and ϕM_n are considerably different from the corresponding *Standard Specifications* results.

Design Requirement 2

Assume that the strand development length = 7 ft for bonded strands and 14 ft for debonded strands. Determine the envelope of the flexural capacity along the span length. Assume 12 of the 46 strands are debonded as shown in **Figure 3-5**. Note that even though 14 ft is a very conservative estimate of development length, it has little impact on the flexural strength of the member.

3.5.2 Comparative Results

Table 3-3 compares results of the strain compatibility approach for f_c' = 5.8 ksi and 8.5 ksi with the results of flexural design of the example of Section 3.5.1 using both the *Standard Specifications* and the *LRFD Specifications*.

Note that the comparisons are for an untopped box beam and not the beam with deck slab in the preceeding section.

	f_c' = 5.8 ksi			f_c' = 8.5 ksi		
	STD Spec.	LRFD Spec.	Strain Comp.	STD Spec.	LRFD Spec.	Strain Comp.
Neutral axis depth, c, in.	9.83	21.40	16.39	7.97	14.89	8.12
Compression block depth, a, in.	7.47	16.26	12.46	5.18	9.68	5.28
Steel stress at ultimate flexure, ksi	251	225	240	255	239	260
ϕM_n ft-kips	4,301	4,301	4,505	5,009	4,557	5,106
	95%	95%	100%	98%	89%	100%

Table 3-3 (PCI 8.2.2.6.2-1) Flexural Capacity Prediction by Various Methods

The table clearly shows the advantage of using the accurate strain compatibility approach. For f_c' = 5.8 ksi, the approximate approach utilizes an equation that is not even a function of the steel provided. For f'_c = 8.5 ksi, the *Standard Specifications* give results that are much closer to the strain compatibility approach than the results of the *LRFD Specifications*. Part of the reason is the estimation of the neutral axis depth which is excessive, resulting in a low steel stress and a correspondingly low ϕM_n.

Some designers compound the errors resulting from the approximate procedures by lumping all pretensioning steel in a section into a single location for the purpose of establishing the effective depth. This is incorrect. Only the reinforcement near the tension face of the member should be considered in determining the steel stress using Eqs. [STD 9-17] and [LRFD 5.7.3.1.1-1].

3.6 Flexural Design Example of Negative Moment Regions

3.6.1 Strength Design

Where continuity at interior supports under live load and composite dead loads is desired at interior support, negative moment reinforcement may be provided within the cast-in-place deck slab. The negative moment section is designed as a reinforced section using the compressive strength of the beam concrete regardless of the strength of the cast-in-place concrete.

Use the width of the bottom flange as the width of the concrete compressive stress block, b. Determine the required steel in the deck to resist the total factored negative moment, assuming that the compression block is uniform:

$$R_n = \frac{M_u}{\phi b d^2} \qquad \text{(PCI Eq. 8.2.3.1-1)}$$

where

R_n = strength design factor
M_u = total factored negative moment
d = distance from extreme compression fiber to centroid of the negative moment reinforcing for precast beam bridges made continuous
ϕ = strength reduction factor = 0.9
This value is consistent with cast-in-place concrete construction, rather than ϕ = 1.0 for precast members. This is reasonable as the main reinforcement is placed in the field.

Estimate the required area of steel using the following equation:

$$\rho = \frac{1}{m}\left[1 - \sqrt{1 - \frac{2mR_n}{f_y}}\right]$$ (PCI Eq. 8.2.3.1-2)

where

$$m = \frac{f_y}{0.85 f_c'}$$ (PCI Eq. 8.2.3.1-3)

f_y = yield stress of non-prestressed conventional reinforcement
f_c' = compressive concrete strength at 28 days for the beam

The steel area, $A_s = \rho bd$. Alternatively, A_s may be determined using one of several approximate methods. For example,

$$A_s \cong \frac{M_u}{0.9\phi f_y d}$$ (PCI Eq. 8.2.3.1-4)

The above equation implies that the lever arm between the tension and compression stress resultants is approximately 0.9d.

The design moment strength, ϕM_n, may be computed by:

$$\phi M_n = \phi A_s f_y \left(d - \frac{a}{2}\right)$$ (PCI STD Eq. 8-16)

where

a = depth of compression block = $\frac{A_s f_y}{0.85 f_c' b}$ (PCI STD Eq. 8-17)
A_s = area of nonprestressed tension reinforcement

If the depth of the compression block is larger than the thickness of the bottom flange, flanged section analysis similar to that used for the positive moment section will need to be done.

3.6.2 Standard Specifications Reinforcement Limits

Maximum reinforcement

$$\rho_b = \frac{0.85\beta_1 f_c'}{f_y}\left(\frac{87{,}000}{87{,}000 + f_y}\right)$$ (STD Eq. 8-18)

$$\rho_{\max} = 0.75\rho_b$$

Minimum reinforcement

The total amount of nonprestressed reinforcement should be adequate to develop an ultimate moment at the critical section at least 1.2 times the cracking moment. The cracking moment may be calculated as for a prestressed concrete section except $f_{pe} = 0$.

$$\phi M_u \geq 1.2 M_{cr}$$

3.6.3 LRFD Specifications Reinforcement Limits

Maximum reinforcement

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Minimum reinforcement

$$A_s \geq \frac{f'_c}{f_y} bd \qquad \text{(LRFD Eq. 5.7.3.3-2)}$$

3.6.4 Serviceability

The deck slab is not prestressed and therefore is not subjected to the tensile stress limits specified under service load conditions for prestressed concrete members. Distribution of the flexural reinforcement in the deck slab should be checked in order to control cracking. The best crack control is obtained when the steel reinforcement is well distributed over the zone of maximum concrete tension [STD 8.17.2.1]. Several bars at moderate spacing are more effective in controlling cracking than one or two larger bars of equivalent area. Crack width is controlled by:

- steel stress
- thickness of concrete cover
- area of concrete surrounding each individual reinforcing bar
- surface condition of the reinforcing bars

The *Standard Specifications* and the *LRFD Specifications* use the same approach for crack control. The tensile stress in the mild reinforcement at service loads, f_s or f_{sa}, should not exceed:

$$f_s = \frac{z}{(d_c A)^{1/3}} \leq 0.6 f_y \qquad \text{(STD Eq. 8-61)}$$

$$f_{sa} = \frac{Z}{(d_c A)^{1/3}} \leq 0.6 f_y \qquad \text{(LRFD Eq. 5.7.3.4-1)}$$

where

d_c = depth of concrete from extreme tension fiber to center of bar

A = area of concrete having the same centroid as the tensile reinforcement and bounded by the surfaces of the cross-section and a straight line parallel to the neutral axis, divided by the number of bars

Z = z = crack width parameter

In situations where the concrete surface is subject to severe exposure conditions, a maximum value of Z = 130 kip/in. is used in design. For moderate exposure conditions, a maximum value of Z = 170 kip/in. is used.

The spacing s of mild steel reinforcement in the layer closest to the tension face shall satisfy the following:

$$s \leq \frac{700\gamma_e}{\beta_s f_{ss}} - 2d_c \qquad (5.7.3.4\text{-}1)$$

in which:

$$\beta_s = 1 + \frac{d_c}{0.7(h - d_c)}$$

3.6.5 Fatigue

The longitudinal deck reinforcement in the negative moment zone over the piers must be checked for fatigue. This portion of the deck is likely to crack due to service loads and the steel stress range may be significant. The stress range in reinforcement is limited by:

$$f_f = 21 - 0.33 f_{\min} + 8\left(\frac{r}{h}\right) \qquad \text{(STD Eq. 8-60, LRFD Eq. 5.5.3.2-1)}$$

where

f_f = stress range

f_{min} = algebraic minimum stress level, positive if tension, negative if compression
r/h = ratio of base radius to height of rolled-on transverse deformations; if the actual value is not known, 0.3 may be used.

For stress calculation according to the *LRFD Specifications*, the special fatigue truck loading must be introduced to the continuous structure.

The stress range in straight reinforcement and welded wire reinforcement without a cross weld in the high-stress region resulting from the fatigue load combination, specified in Table 3.4.1-1, shall satisfy:

$$f_f \leq 24 - 0.33 f_{min} \quad (5.5.3.2\text{-}1)$$

The stress range in straight welded wire reinforcement with a cross weld in the high-stress region resulting from the fatigue load combination, specified in Table 3.4.1-1, shall satisfy:

$$f_f \leq 16 - 0.33 f_{\min} \quad (5.5.3.2\text{-}2)$$

3.7 Shear

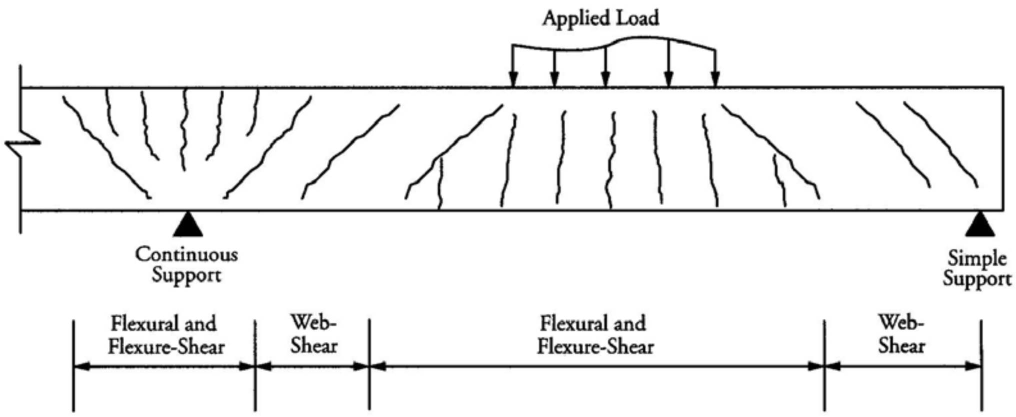

Figure 3-4 (Figure 8.4.1-) Types of Cracking in Concrete Beams

3.7.1 LRFD Specifications

There are two methods of shear design presented in the *LRFD Specifications*. The most general method is the strut-and-tie model. This model can be applied to any design situation, including members with irregular cross-sections or discontinuities. It is also used to design a member for all load effects, not just shear.

The method used for typical shear design is the sectional design model, or modified compression field theory developed by Collins, Mitchell and others. This method is based on the variable angle truss model in which the inclination of the diagonal compression field is allowed to vary. This differs from the approach used in the *Standard Specifications* in which this angle is always assumed to be 45E. This is especially significant for prestressed concrete members where the inclination is typically 20E to 40E degrees due to the effect of the prestressing force.

This model also differs from the shear design method found in the *Standard Specifications* because the concrete contribution, V_c, is attributed to tension being carried across the compression diagonals. The contribution has been determined experimentally and has been related to the strain in the tension side of the member. In general, the higher the strain in the tension side at ultimate, the wider the shear cracks, and in turn the smaller the concrete contribution.

It is significant to note that the concrete contribution, V_c, is what sets the sectional design model apart from the strut-and-tie model. Both models are based on the variable-angle truss analogy in which a concrete member resists loads by a truss composed of concrete "compression struts" and steel "tension ties." While this model is an effective tool in estimating the shear capacity of concrete members, it has been found to underestimate V_c when compared to test results. Therefore, the sectional design method can be expected to give higher capacities than the strut-and-tie model.

- LRFD 5.8.2.4 Regions required transverse reinforcement:

$$V_u > 0.5\Phi (V_c + V_p)$$ (LRFD Eq. 5.8.2.4-1)

- LRFD 5.8.2.5 Minimum transverse reinforcement:

For segmental post-tensioned concrete box girder bridges:

$$A_v \geq 0.05 \frac{b_w s}{f_y}$$ (LRFD Eq. 5.8.2.5-2)

Except for segmental post-tensioned concrete box girder bridge:

$$A_v \geq 0.0316\sqrt{f_c'}\frac{b_v s}{f_y}$$ (LRFD Eq. 5.8.2.5-1)

- LRFD 5.8.2.7 Maximum spacing of transverse reinforcement:

If $v_u < 0.125f'_c$, then $s_{max} = 0.8d_v \leq 24$ inch (LRFD Eq. 5.8.2.7-1)

If $v_u \geq 0.125f'_c$, then $s_{max} = 0.4d_v \leq 24$ inch (LRFD Eq. 5.8.2.7-2)

- LRFD 5.8.2.9 Shear Stress on Concrete:

$$v_u = \frac{\left|V_u - \varphi V_p\right|}{\phi b_v d_v}$$ (LRFD Eq. 5.8.2.9-1)

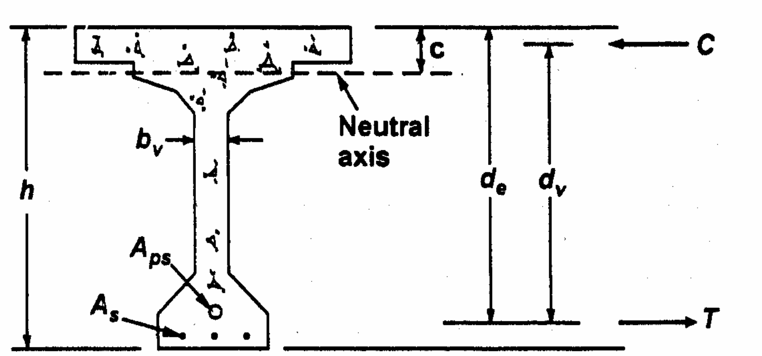

Figure C5.8.2.9-1 - Illustration of the Terms b_v and d_v.

Figure 3-5

The *LRFD Specifications*, Article 5.8.3 introduces the sectional design model. Subsections 1 and 2 describe the applicable geometry required to use this technique to design web reinforcement.

The nominal resistance is taken the lesser of:

$$V_n = V_c + V_s + V_p \text{, or,} \qquad \text{(LRFD Eq. 5.8.3.3-1)}$$

$$V_n = 0.25 f'_c b_v d_v + V_p \qquad \text{(LRFD Eq. 5.8.3.3-2)}$$

where

b_v = effective web width

d_v = effective shear depth

LRFD Eq. (5.8.3.3-2) represents an upper limit of V_n to assure that the concrete in the web will not crush prior to yield of the transverse reinforcement.

The *LRFD Specifications* defines the concrete contribution as the nominal shear resistance provided by the tensile stresses in the concrete. This resistance is computed using the following equation:

$$V_c = 0.0316 \beta \sqrt{f'_c} b_v d_v \qquad \text{(LRFD Eq. 5.8.3.3-3)}$$

The units used in the *LRFD Specifications* are kips and inches. The factor 0.0316 is equal to

$$\frac{1}{\sqrt{1{,}000}}$$

which coverts the expression from psi to ksi units for the concrete compressive strength.

The contribution of the web reinforcement is given by the general equation:

$$V_s = \frac{A_v f_y d_v (\cot\theta + \cot\alpha)\sin\alpha}{s} \qquad \text{(LRFD Eq. 5.8.3.3-4)}$$

where the angles, θ and α , represent the inclination of the diagonal compressive stresses measured from the horizontal beam axis and the angle of inclination of transverse reinforcement to longitudinal axis..

For cases of vertical web reinforcement, the expression for V_s simplifies to:

$$V_s = \frac{A_v f_y d_v \cot\theta}{s} \qquad \text{(LRFD Eq. C5.8.3.3-1)}$$

Transverse shear reinforcement should be provided when:

$$V_u > 0.5\phi\left(V_c + V_p\right) \qquad \text{(LRFD Eq. 5.8.2.4-1)}$$

Where the reaction force in the direction of the applied shear introduces compression into the end region of a member, the location of the critical section for shear shall be taken as d_v from the internal face of the support

.

To determine the nominal resistance, the design engineer must determine θ and β from the *LRFD Specifications*, Article 5.8.3.4. For mildly reinforced concrete sections, the values of θ and β are β and 45E respectively. These will produce results similar to the *Standard Specifications*. However, for prestressed concrete, the engineer can take advantage of the precompression and use lower angles of β, which optimizes the web reinforcement.

3.7.2 Design Procedure

To design the member for shear, the designer first determines the factored shear due to applied loads at the section under investigation. The critical section is located at d_v,
or $0.5d_v\cot\theta$. The value for d_v is generally taken from midspan flexural capacity calculations, where $d_v = d - a/2$. The shear contribution from any harped strand, V_p, is then computed.

Unless more accurate calculations are made, ε_x shall be determined as:

- If the section contains at least the minimum transverse reinforcement as specified in LRFD Art. 5.8.2.5:

$$\varepsilon_x = \frac{\frac{|M_u|}{d_v} + 0.5N_u + 0.5|V_u - V_p|\cot\theta - A_{ps}f_{po}}{2(E_sA_s + E_pA_{ps})} \le 0.001 \qquad \text{(LRFD Eq. 5.8.3.4.2-1)}$$

- If the section contains less than the minimum transverse reinforcement as specified in LRFD Art. 5.8.2.5:

$$\varepsilon_x = \frac{\frac{|M_u|}{d_v} + 0.5N_u + 0.5|V_u - V_p|\cot\theta - A_{ps}f_{po}}{E_sA_s + E_pA_{ps}} \le 0.002 \qquad \text{(LRFD Eq. 5.8.3.4.2-2)}$$

- If the value of ε_x from Equations 1 or 2 is negative, the strain shall be taken as:

$$\varepsilon_x = \frac{\frac{|M_u|}{d_v} + 0.5N_u + 0.5|V_u - V_p|\cot\theta - A_{ps}f_{po}}{2(E_cA_c + E_sA_s + E_pA_{ps})} \qquad \text{(LRFD Eq. 5.8.3.4.2-3)}$$

The specifications indicate that the area of prestressing steel, A_{ps}, must account for lack of development near the ends of prestressed beams. Any mild reinforcement or strand in the compression zone of the member, which is takenas one-half of the overall depth (h/2), should be neglected when computing A_s and A_{ps} for use in this calculation. This is very important when evaluating members with harped strand, since near the end of typical beams, harped strands are near the top of the beam. Because of this, it is recommended that the straight and harped strands be considered separately in the analysis. It is the physical location of each strand that is important and not the centroid of the group.

The variable, f_{po}, represents the stress in the prestressing strand when the stress in the surrounding concrete is zero. For the usual level of prestressing, $0.7f_{pu}$ may be used.

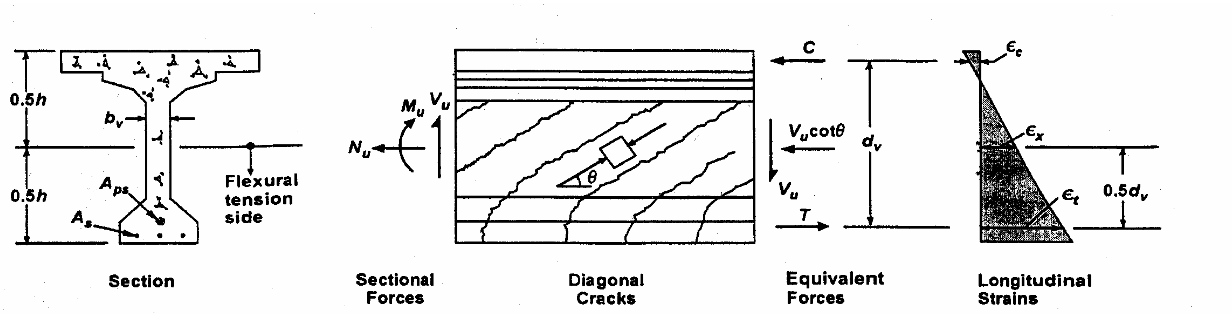

Figure 5.8.3.4.2-1 - Illustration of Shear Parameters for Section Containing at Least the Minimum Amount of Transverse Reinforcement, V_p=0.

Figure 3-6

The value of β corresponding to v/f'_c and ε_x is compared to the assumed value of 2. If the values match, V_c is calculated using Eq. (5.8.3.3-3) which the value of θ from the table. If they do not match, the value of β taken from the table is used for another iteration. Of the quantities computed thus far, only ε_x will change with a new value for β, so the effort required for additional iterations is minor.

After V_c has been computed, V_s is calculated using Eq. (5.8.3.3-1). The quantity of shear reinforcement is then calculated using Eq. (C5.8.3.3-1) with the value of β from the table. After determining the amount of shear reinforcement needed, the designer should check the maximum spacing allowed by the specifications as given in Article 5.8.2.7. Also, the amount of shear reinforcement should be checked to ensure that it is equal to or larger than the minimum value required by the specifications, which is:

$$A_v = 0.0316\sqrt{f'_c}\,\frac{b_v s}{f_y} \qquad \text{(LRFD Eq. 5.8.2.5-1)}$$

In regions of high shear stresses, the longitudinal (flexural) reinforcement must also be able to carry the additional stress due to shear, i.e., the horizontal component of the diagonal compression field. Therefore, the amount and development of the longitudinal reinforcement must satisfy Eq. (5.8.3.5-1):

$$A_s f_y + A_{ps} f_{ps} \geq \left[\frac{|M_u|}{d_v \phi} + 0.5\frac{N_u}{\phi} + \left(\left|\frac{V_u}{\phi} - V_p\right| - 0.5V_s\right)\cot\theta\right] \qquad \text{(LRFD Eq. 5.8.3.5-1)}$$

Satisfying this equation is very important for prestressed concrete beams, especially near non-continuous supports where a substantial portion of the prestressing strands are harped and the transfer length of the strand extends into the span. Harped strands are not effective in contributing to this longitudinal reinforcement requirement since they are above midheight of the member.

The *LRFD Specifications* recommend that this criterion be checked at the face of the bearing. At this section, which usually lies within the transfer length of the strands, the effective prestressing force in the strands is not fully developed. Thus, the term f_{ps} should be calculated as a portion of the effective prestress force based on linear variation starting from zero at the end of the beam to full effective prestress at the transfer length. The designer should not be confused by the term f_{ps}, which generally refers to the prestress force at Strength Limit State, because the strands at this section do not have enough development length to provide such level of prestress. If the strands are well anchored in a diaphragm at the end of the member, the stress in the strands, f_{ps}, can be considered to equal the stress in the strands at Strength Limit State. This approach of varying f_{ps} to account for lack of development is preferred over the method implied by the definition of A_{ps} for the lack of development.

3.8 Horizontal Interface Shear – Shear Friction

3.8.1 Theory

Cast-in-place concrete decks designed to act compositely with precast concrete beams must be able to resist the horizontal shearing forces at the interface between the two elements. The basic strength equation for the design of the interface between the deck and beam is:

$$V_u \leq \phi V_{nh} \qquad \text{(STD Eq. 9-31a)}$$

where

V_u = factored shear force acting on the interface

ϕ = strength reduction factor

V_{nh} = nominal shear capacity of the interface

Design is carried out at various locations along the span, similar to vertical shear design. Theoretical calculation of the shearing force acting on the interface at a given section is not simple because the section does not behave as a linear elastic material near ultimate capacity. If it did, the shear stress, horizontal or vertical, at any fiber in a cross-section would be calculated from the familiar equation:

$$v_h = \frac{VQ}{Ib} \qquad \text{(PCI Eq. 8.5.1-1)}$$

Where

V = vertical shear force at the section

I = moment of inertia

b = section width at the fiber being considered

Q = first moment of the area above (or below) the fiber being considered

However, at ultimate conditions, the material is no longer elastic and the concrete may be cracked at the section being considered. Further, the composite cross-section consists of two different types of concrete with different properties. Therefore, application of the above equation to design at ultimate, without modification, would yield questionable results.

Loov and Patnaik (1994) determined that the above equation may yield adequate results if both the cracked section moment of inertia and area moment of a transformed composite section are used. The section would be transformed using the slab-to-beam modular ratio used in flexural design by the allowable stress method. However, this approach is still too complicated. It confuses the calculations at two limit states: service and ultimate.

Kamel (1996) used equilibrium of forces to show that:

$$v_h = V/(jd)b_v \qquad \text{(PCI Eq. 8.5.1-2)}$$

where

V = factored vertical shear at the section in question

d = effective depth of the member

jd = distance between the tension and compression resultant stresses in the section. This is the same distance as d_v used in the *LRFD Specifications*.

b_v = section width at the interface between the precast and the cast-in-place concrete. It is important to understand that b_v is not the web width.

Another important issue is which loads should be used to calculate V_u at a section. Neither the *Standard Specifications* nor the *LRFD Specifications* give guidance in this regard. While most designers would use all loads to compute V_u, a strong case can be made for excluding the self-weight of the precast concrete member, and the weight of the deck since they are present prior to composite action taking effect. Some designers and agencies, such as the Illinois Department of Transportation, use only the composite loads, which include the superimposed dead loads (barriers, wearing surface, etc.) and the live loads. Fortunately, the amount of reinforcement required, even with consideration of all loads, is reasonable in practical applications.

To determine the shear capacity of the interface, the *LRFD Specifications* use a form of the well-established shear friction theory, while the *Standard Specifications* use an empirical approach based on several investigations, for example, Birkeland and Birkeland (1966), Mast (1968), Kriz and Raths (1965) and Hofbeck, et al (1969).

It is not possible to directly compare the results of the two specifications because the method used in the *Standard Specifications* is stated in terms of vertical shear while in the LRFD Specifications is stated in terms of horizontal (interface) shear.

3.8.2 LRFD Specifications

LRFD Specifications give no guidance for computing horizontal shear due to factored loads. The following formula may be used as discussed in Section 3.8.1 with the substitution d_v for jd:

$$v_{uh} = \frac{V_u}{d_v b_v} \qquad \text{(PCI Eq. 8.5.3-1)}$$

where

v_{uh} = horizontal factored shear force per unit area of interface

V_u = factored vertical shear force at specified section due to superimposed loads

d_v = the distance between resultants of tensile and compressive forces =

b_v = interface width

Required strength □ nominal strength, or:

$$v_{uh} A_{cv} \leq \phi V_n \qquad \text{(PCI Eq. 8.5.3-2)}$$

where V_n = nominal shear resistance of the interface surface

$= cA_{cv} + \mu \left[A_{vf} f_y + P_c \right]$

where

c = cohesion factor = 0.10 for this case

μ = friction factor = 1.0 for this case

A_{cv} = interface area of concrete engaged in shear transfer

A_{vf} = area of shear reinforcement crossing the shear plane within area

P_c = permanent net compressive force normal to the shear plane (may be conservatively neglected)

f_y = yield strength of shear reinforcement

Typically, the top surface of the beam is intentionally roughened to an amplitude of 1/4 in.

Therefore, for normal weight concrete cast against hardened, roughened, normal weight concrete, the above relationships may be reduced to the following formula:

$$v_{uh} \leq \phi \left(0.1 + A_{vf} f_y / A_{cv} \right) \qquad \text{(PCI Eq. 8.5.3-3)}$$

where the minimum $A_{vf} = \left(0.05 b_v s \right) / f_y$ (LRFD Eq. 5.8.4.1-4)

Nominal shear resistance is the lesser of:

$$V_n \le K_1 f'_c A_{cv}\text{, and,} \qquad \text{(LRFD Eq. 5.8.4.1-2)}$$

$$V_n \le K_2 A_{cv} \qquad \text{(LRFD Eq. 5.8.4.1-3)}$$

While the *LRFD Specifications* require that minimum reinforcement be provided regardless of the stress level at the interface, designers may choose to limit this reinforcement to cases where v_{uh} / ϕ is greater than 0.10 ksi. This would be consistent with the *Standard Specifications*, the ACI Code and other references. It would seem to be impractical and an unnecessary expense to provide connectors in a number of common applications, such as precast stay-in-place panels if the interface stress is lower than 0.10 ksi.

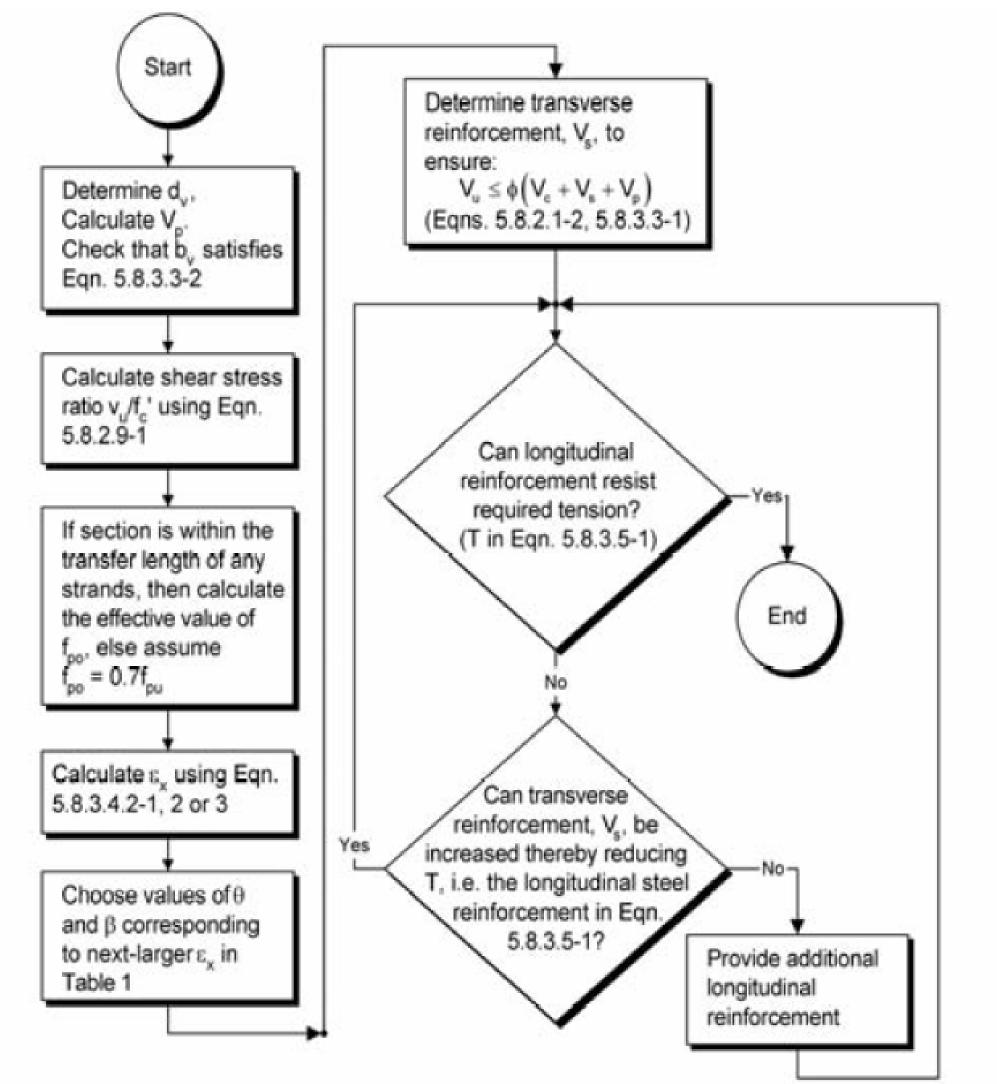

Figure C5.8.3.4.2-5 Flow Chart for Shear Design of Section Containing at Least Minimum Transverse Reinforcement.

Figure 3-7 (LRFD Figure C5.8.3.4.2-1) Flow Chart for Shear Design

Table 5.8.3.4.2-1 Values of θ and β for Sections with Transverse Reinforcement.

$\frac{v_u}{f'_c}$	$\varepsilon_x \times 1{,}000$								
	≤−0.20	≤−0.10	≤−0.05	≤0	≤0.125	≤0.25	≤0.50	≤0.75	≤1.00
≤0.075	22.3 6.32	20.4 4.75	21.0 4.10	21.8 3.75	24.3 3.24	26.6 2.94	30.5 2.59	33.7 2.38	36.4 2.23
≤0.100	18.1 3.79	20.4 3.38	21.4 3.24	22.5 3.14	24.9 2.91	27.1 2.75	30.8 2.50	34.0 2.32	36.7 2.18
≤0.125	19.9 3.18	21.9 2.99	22.8 2.94	23.7 2.87	25.9 2.74	27.9 2.62	31.4 2.42	34.4 2.26	37.0 2.13
≤0.150	21.6 2.88	23.3 2.79	24.2 2.78	25.0 2.72	26.9 2.60	28.8 2.52	32.1 2.36	34.9 2.21	37.3 2.08
≤0.175	23.2 2.73	24.7 2.66	25.5 2.65	26.2 2.60	28.0 2.52	29.7 2.44	32.7 2.28	35.2 2.14	36.8 1.96
≤0.200	24.7 2.63	26.1 2.59	26.7 2.52	27.4 2.51	29.0 2.43	30.6 2.37	32.8 2.14	34.5 1.94	36.1 1.79
≤0.225	26.1 2.53	27.3 2.45	27.9 2.42	28.5 2.40	30.0 2.34	30.8 2.14	32.3 1.86	34.0 1.73	35.7 1.64
≤0.250	27.5 2.39	28.6 2.39	29.1 2.33	29.7 2.33	30.6 2.12	31.3 1.93	32.8 1.70	34.3 1.58	35.8 1.50

Table 5.8.3.4.2-2 Values of θ and β for Sections with Less than Minimum Transverse Reinforcement.

s_{xe} (in.)	$\varepsilon_x \times 1000$										
	≤−0.20	≤−0.10	≤−0.05	≤0	≤0.125	≤0.25	≤0.50	≤0.75	≤1.00	≤1.50	≤2.00
≤5	25.4 6.36	25.5 6.06	25.9 5.56	26.4 5.15	27.7 4.41	28.9 3.91	30.9 3.26	32.4 2.86	33.7 2.58	35.6 2.21	37.2 1.96
≤10	27.6 5.78	27.6 5.78	28.3 5.38	29.3 4.89	31.6 4.05	33.5 3.52	36.3 2.88	38.4 2.50	40.1 2.23	42.7 1.88	44.7 1.65
≤15	29.5 5.34	29.5 5.34	29.7 5.27	31.1 4.73	34.1 3.82	36.5 3.28	39.9 2.64	42.4 2.26	44.4 2.01	47.4 1.68	49.7 1.46
≤20	31.2 4.99	31.2 4.99	31.2 4.99	32.3 4.61	36.0 3.65	38.8 3.09	42.7 2.46	45.5 2.09	47.6 1.85	50.9 1.52	53.4 1.31
≤30	34.1 4.46	34.1 4.46	34.1 4.46	34.2 4.43	38.9 3.39	42.3 2.82	46.9 2.19	50.1 1.84	52.6 1.60	56.3 1.30	59.0 1.10
≤40	36.6 4.06	36.6 4.06	36.6 4.06	36.6 4.06	41.2 3.20	45.0 2.62	50.2 2.00	53.7 1.66	56.3 1.43	60.2 1.14	63.0 0.95
≤60	40.8 3.50	40.8 3.50	40.8 3.50	40.8 3.50	44.5 2.92	49.2 2.32	55.1 1.72	58.9 1.40	61.8 1.18	65.8 0.92	68.6 0.75
≤80	44.3 3.10	44.3 3.10	44.3 3.10	44.3 3.10	47.1 2.71	52.3 2.11	58.7 1.52	62.8 1.21	65.7 1.01	69.7 0.76	72.4 0.62